1.2.1 Types of DTS (cont..)

e) Linear time invariant (LTI) system.

iff $x[n] \rightarrow y[n]$ then for LTI

$$x[n-k] \rightarrow y[n-k]$$

That means shift or delay in input seq causes a corresponding shift in output seq for an LTI system.

f) Time varing system

Opposite of LTI

g) causal vs. non-causal

For causal system, $y[n] = f(x[n-n_d])$ when $n_d \geq 0$

$$y[n] = f(x[n], x[n-1], \ldots)$$

This means that a causal signal is dependent on present and preceding inputs. The future values has no bearing on the o/p.

For non-causal, the o/p may be dependent on future values.

n) Stable vs. Unstable system.

For a stable system, a bounded input with produce a bounded o/p.

EX Show that the system defined by:

$$y[n] = x[n-2] + 2x[n-3]$$

SOL

$$ax_1[n] = ax_1[n-2] + a2x_1[n-3]$$
$$= ay_1[n]$$

$$bx_2[n] = bx_2[n-2] + b2x_2[n-3]$$
$$= by_2[n]$$

$$ax_1[n] + bx_2[n] = ax_1[n-2] + a2x_1[n-3] + bx_2[n-2] + b2x_2[n-3]$$

$$= ay_1[n] + by_2[n]$$

$\therefore$ system is linear.

EX test whether the following system is time variant or not?

$$y[n] = x[Mn]$$

SOL

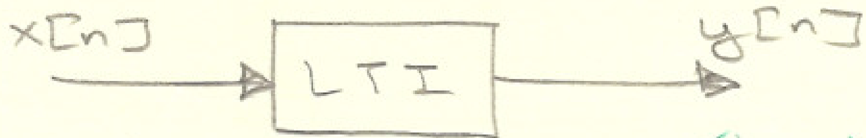
$$y[n-n_0] = x[M(n-n_0)] = x[Mn - Mn_0]$$

$\neq x[Mn - n_0]$ $\therefore$ time varying.

Note: $y[n] = x[Mn]$; $x[n]$ is downsampled

$y[n] = x\left[\frac{n}{M}\right]$; $x[n]$ is highly sampled.

1.3 repersentation of LTI



(impulse) $\delta[n]$ (impulse response) $h[n]$

$\delta[n-k]$ $h[n-k]$

Any sequence $x[n]$ can be represented as

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

$$\therefore\ y[n] = T\{x[n]\}$$

$$= T\left\{\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]\right\}$$

$$= \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

now let $m = n-k$

$$\therefore \quad y[n] = \sum_{m=-\infty}^{\infty} x[n-m]\, h[m]$$

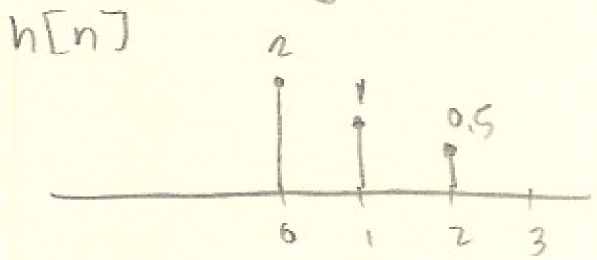
$$= \sum_{k=-\infty}^{\infty} x[n-k]\, h[k]$$

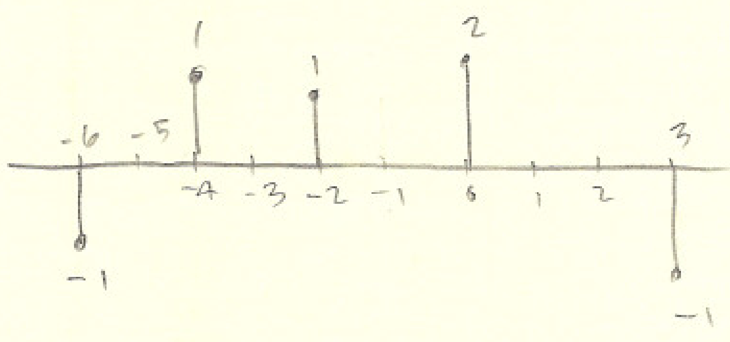
Very similiar to convolution

$$y[n] = x[n] * x[n]$$

The significance... if the input responce of an LTI system is known we can calculate the o/p sequence $y[n]$ for any input sequence.

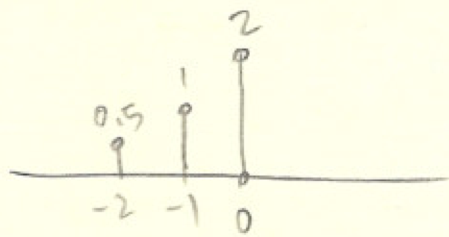
EX The impulse responce of an LTI system is shown below find $y[n]$ for the following seqence $x[n]$





SOL There are 5 methods for solving this.

h[-k]



And then shift through like in communications
Another method.

$$y[n] = \sum_k x[k]\, h[n-k]$$

$$y[n] = T \{ x[-6] + x[-4] + x[-2] + x[0] + x[3] \}$$